OPTIMUM ELASTIC DESIGN OF GRID ROOFS

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In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

by
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to the

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MARCH, 1977

To

my mother Nalini

and uncle Dattatray alias Aba Kanitkar

\- Kumar

CE-1977-M-KUL-OPT

CERTIFICATE

This is to certify that the thesis entitled "OPTIMUM ELASTIC DESIGN OF GRID ROOFS" by Ganesh Kulkarni is a record of work carried out under my supervision and has not been submitted elsewhere for a degree.

Kapoor 22/2/77

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Ganesh Kulkarni.

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ABSTRACT

In the present work optimization of reinforced concrete elastic grid roofs is carried out. The objective function chosen is the total cost of materials with constraints on maximum shear stress and maximum compression reinforcement in the sections. The problem is formulated as a mathematical programming problem and is solved using sequential unconstrained minimization technique with interior penalty function approach. Variable metric algorithm is used for unconstrained minimization.

The grid structure is analysed by stiffness method of structural analysis and the resulting equations are solved by Choleski's decomposition method.

Parametric study is made by varying the number of beams in the grid. An empirical formula for obtaining the optimum number of beams is suggested.

CHAPTER 1

INTRODUCTION

1.1 GENERAL

Concrete and steel are widely used materials in civil engineering construction works. With the advent of digital computers and development of optimization techniques substantial saving of these construction materials can be achieved. Present work attempts to obtain the minimum cost design of large elastic reinforced concrete grid system via mathematical programming.

Designers are sometimes called upon to provide designs to cover up large floor areas with the following conditions.

1) The entire floor area to be clear of any structural obstruction

2) The upper floor to be flat and

3) The head-room to be maximum.

The first condition can be achieved by eliminating intermediate columns and providing deep main beams. This solution does not fall in line with the third condition but can be improved upon by the use of inverted beams. This again does not satisfy the second condition.

The solution to this problem is the use of grid system, constructed of reinforced cement concrete beams with thin slab on top of it.

In a grid system there are usually two sets of beams running in two different directions and are rigidly joined at the points of intersection. In general there can be more than two sets, their respective elements need not be parallel and the angles of intersection are arbitrary. However, this general form of grid is rarely encountered in practice. The grid systems normally used are orthogrid and diagrid which are shown in Figures 1.1 and 1.2 respectively. In orthogrid there are two sets of beams running parallel to the supports and which intersect each other at right angles. In diagrid system the parallel beams running in two directions are inclined to the spans. The angles of inclination of the two sets with the spans need not be same.

The grid system is a statically indeterminate structure and the degree of redundancy depends upon the number of beams used in each direction. The load on the grid is essentially the dead load and the live load. The load shared by a beam will depend upon the spacing of the beams. Thus the spacing of the beams becomes an important parameter from the point of view of the designer who is always conscious to provide a minimum cost design. Keeping in mind

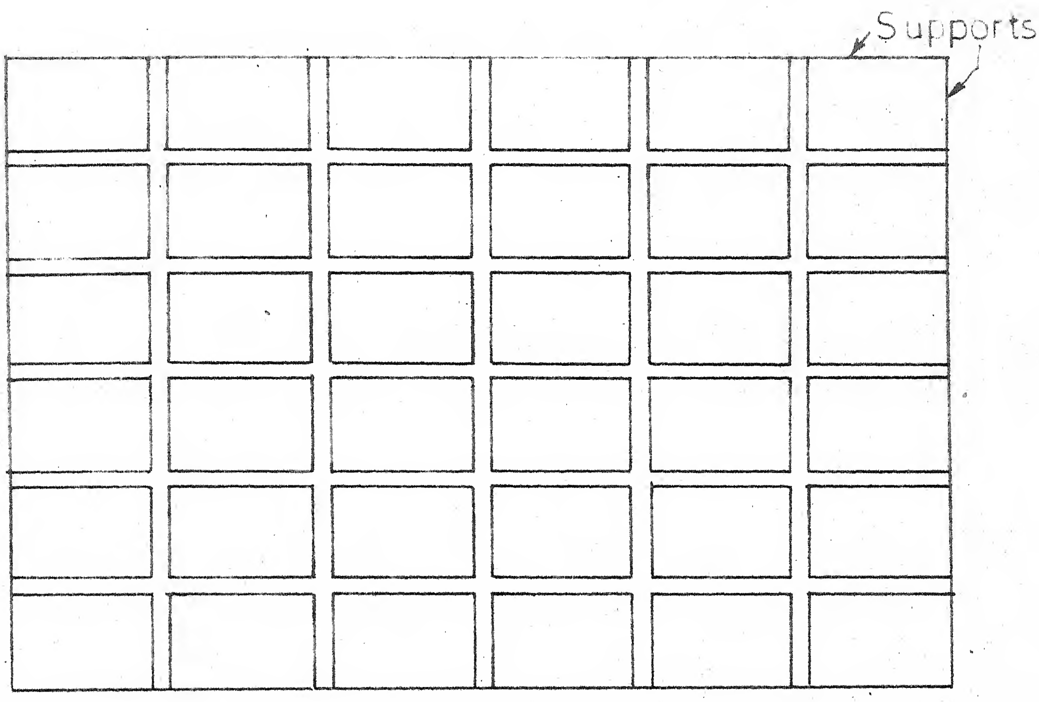

FIG.1.1 ORTHOGRID

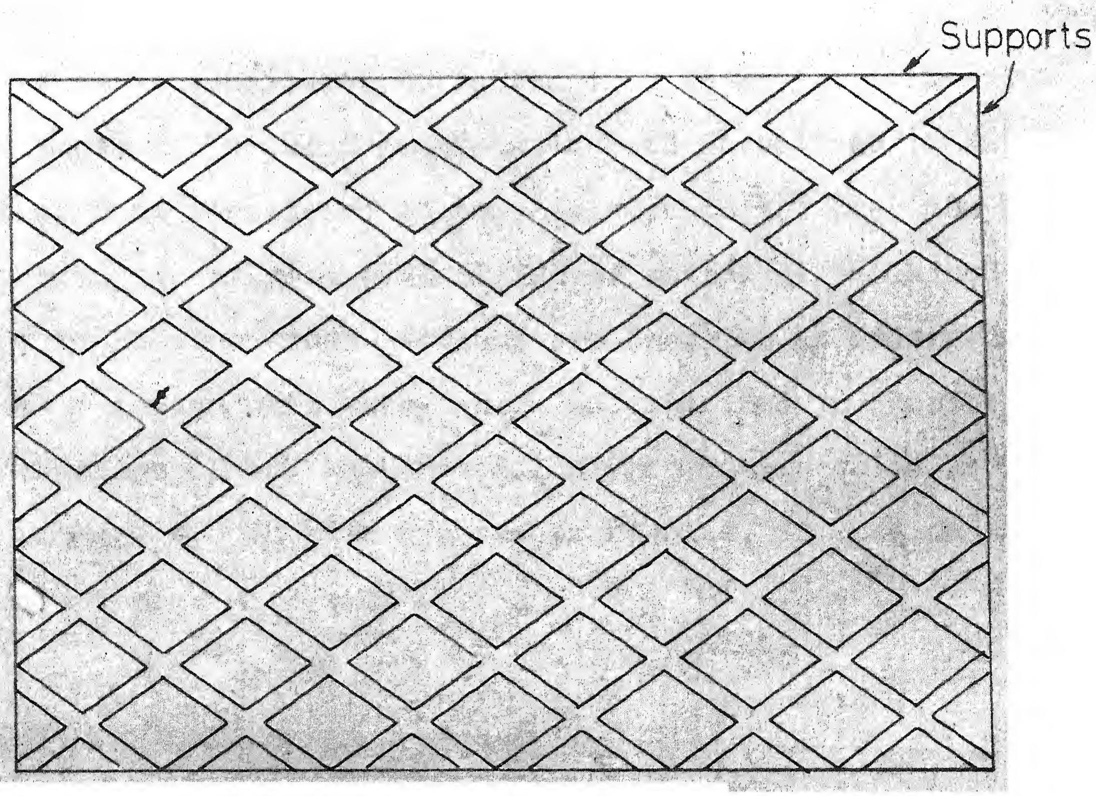

FIG.1.2 DIAGRID

that the grid system is highly indeterminate and the objectiv of the designer to have the minimum cost design, this problem is most suitable to be formulated as optimum design problem, the solution of which can be tracked down with the help of digital computer.

1.2 STATEMENT OF THE PROBLEM

Present investigation is undertaken to find optimum elastic design of reinforced concrete grid roofs. The parametric study for the number of beams leading to the minimum cost design has been made. A grid square in plan with two sets of equally spaced beams, having same spacing in the two directions and running parallel to the supports is considered. The grid is considered to be fixed on supports. The width and depth of the beam sections in both directions is kept same. The slab is considered to be simply supported over the grid. The grid is analysed with stiffness method as explained in Chapter 2. The cross sections are designed according to the elastic design philosophy which is explained in Chapter 3. The formulation of the mathematical programming problem is explained in Chapter 4. Chapter 5 presents results, discussion and conclusions.

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1.3 REVIEW OF RELATED TOPICS

1.3.1 Analysis of Grids

Lazarides (21) used deflection compatibility equations at the intersection of beams to analyse a grid system. This turned out to be a tedious and impracticable method. Moment distribution method was applied for grid analysis by Halasz (13) and also by Ewell, Okubo and Abrams (4). This method also involved lengthy computations. Guyon (12) obtained a solution by representing the grid system with an anisotropic plate where, the members were assumed to have negligible torsional resistance. This solution was generalized by Massonnet (1,23) to take the torsional rigidity into consideration. This equivalent plate method was applied by Taraporewalla (30) for designing grid systems. Hendry and Jaeger (14) proposed to replace the cross-beams of the grid system by a uniformly distributed medium. Here torsional rigidity of beams is taken into account but that of the cross-beams is neglected. The analysis is done by solving the differential equation of loading by harmonic analysis. Lightfoot and Sawko (22) applied generalized slope-deflection method for the analysis of grid system with the help of computer. Reilly (27) incorporated the effect of warping in the analysis of grids by stiffness method. Work in plastic analysis of grids has also been done by Hodge (17), Heyman (15,16) and Grigorian (9,10).

In the present work stiffness method of structural analysis is used to analyse the elastic grid. This method is amenable to handle large redundancy and is simple to programme on computer.

1.3.2 Optimization of Grids

Optimization of simply supported plane steel grillages carrying uniformly distributed loads was studied by Grundy (11). Here weight function was obtained from mid-span bending moment by limiting extreme fibre stresses or ultimate moment or maximum deflection to a permissible value. Assuming the weight function to be differentiable, optimum values of spanswere found out by partial differentiation.

Optimization of a simply supported reinforced concrete square grid roof was studied by Narayanan (24). The analysis was carried out by representing the grid with an equivalent orthotropic plate. The section was designed by plastic method for mid-span bending moment. For the optimal design, section of beam, depth of slab, percentage of steel in beam and slab and spacing of beams were found out.

Ramesh and Karve (26) carried out parametric study for orthogonal grids with different boundary conditions and with varying number of beams. The system was analysed as a stiffened plate adopting a conforming rectangular

plate element and using finite element technique. The parameters studied are, the ratio of span to plate thickness and the ratio of span to depth of beams. Ratio of span to width of beams was taken equal to 24 in all cases. Charts are presented for different grids of given aspect ratio and given number of beams running in each direction. For a given grid the values of the above mentioned parameters, which achieve given effective thickness with minimum volume of material can be found out from the charts.

Keshav Rao (19) used fully stressed design method for finding an optimal layout for grid. In this method starting with a uniform depth of slab the depths at various nodes of the slab are changed continually to obtain fully stressed design in each cycle. After a certain number of iterations the change in nodal depths from one cycle to next cycle becomes negligible. At this stage the ridges obtained by joining the nodes of large depth are interpreted as beams. From this a practical layout of beam system is obtained showing substantial saving of material.

Kwlecinski and Kleiber (20) studied optimization of simply supported rectangular grids composed of two rectilinear families of perfectly plastic bars subjected to transverse load. Grid bars were assumed to be capable of resisting torque. Optimum configuration was found based on minimizing weight of the structure.

The present work studies the optimum spacing of beams leading to the minimum cost design of the reinforced cêment concrete grid system.

1.3.3 Optimization Techniques

Optimization techniques for constrained problems can be broadly classified into two groups. One, direct methods and other, indirect methods. Of the direct methods, Zoutendijk's method (31) of feasible directions is used for problems with non-linear inequality constraints. The gradient projection method by Rosen (28,29) is well used for problems with linear constraints but is ineffective for non-linear constraints. The indirect method of penalty function (8) is simple to apply for the constrained problems. There are two approaches for the penalty function method. One is exterior penalty function method and other is interior penalty function method. The latter has certain advantages over the exterior penalty function method. In the approach by interior penalty function, 1) a feasible design point is always at hand, 2) improved designs are obtained in a sequential process, and 3) the constraints become critical only at the optimum. Hence in the present work interior penalty function approach has been used. This method requires the use of an unconstrained minimization technique.

Steepest descent method was used in early days for unconstrained minimization. But for eccentric functions this technique faces difficulties in convergence. Fletcher and Reeves (7) developed the conjugate gradient technique popularly known as Fletcher-Reeves method. Powell (25) introduced the conjugate direction method in 1964 for unconstrained minimization. Devidon (3) invented the variable metric method to carry out unconstrained optimization. This was further sharpened by Fletcher and Powell (6). The method is also known as DFP method. This technique is powerful and has been successfully used for optimizing highly distorted functions.

For present work Sequential Unconstrained Minimization Technique (SUMT) of Fiacco and McCormick (5) is used which is an interior penalty function method. The DFP algorithm is used as the technique for unconstrained minimization and linear minimization is carried out by polynomial fitting.

CHAPTER 2

ANALYSIS OF GRID

2.1 INTRODUCTION

Stiffness method of structural analysis is used for analysing the grid in the present work. Each beam intersection is considered as a node with three degrees of freedom viz., the transverse deformation and the rotations about two directions. Advantage of the symmetry of structure is taken to reduce the size of the problem. To seek the solution of the stiffness formulation Choleski's decomposition technique is used.

The materials considered to be used for the construction of the grid are: 1) Concrete of M150 grade and 2) The reinforcement to be of Grade I mild steel. The properties of concrete and steel are in accordance with IS-456-1964.

2.2 ASSUMPTIONS

Following assumptions are made in the analysis of grid for the present work.

1) The intersection of the beams i.e. the nodes are rigid joints.

2) Reinforced cement concrete is homogeneous, isotropic and linearly elastic material.

3) The stiffness of beam is obtained from the overall section of the beam neglecting the steel.

4) The value of the coefficient β in the expression for the torsional stiffness ($\beta G b^3 D$) is kept equal to 0.3 for all values of b/D.

5) Poisson's ratio for concrete is assumed equal to zero.

6) Axial deformations in the beams are neglected.

2.3 CONSIDERATION OF SYMMETRY

Only a quadrant of the structure need be analysed by taking the advantage of symmetry about two directions. While solving the simultaneous equations, the condition is imposed that the slopes of beams crossing the line of symmetry are zero at the line of symmetry.

Figures 2.1 and 2.3 show a quarter of the symmetric grid structure when the number of beams running in each directions (Nb) is odd and even respectively. In the case when Nb is odd the lines of symmetry pass along the central beams in two directions. In such case symmetry is taken into account by considering half the stiffnesses of the beams running along the lines of symmetry. When Nb is even

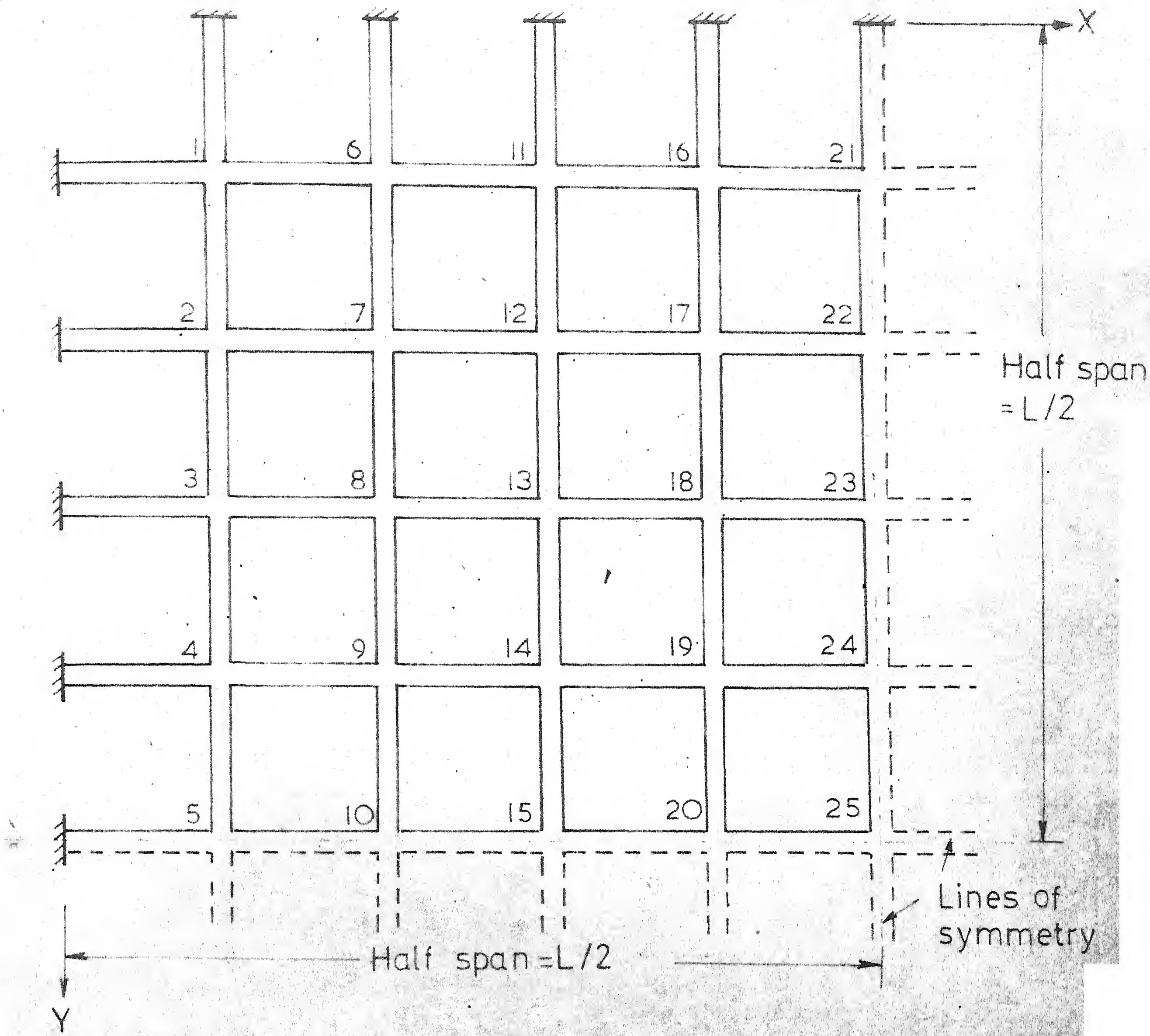


FIG. 2.1 GRID WITH ODD NUMBER OF BEAMS IN EACH DIRECTION

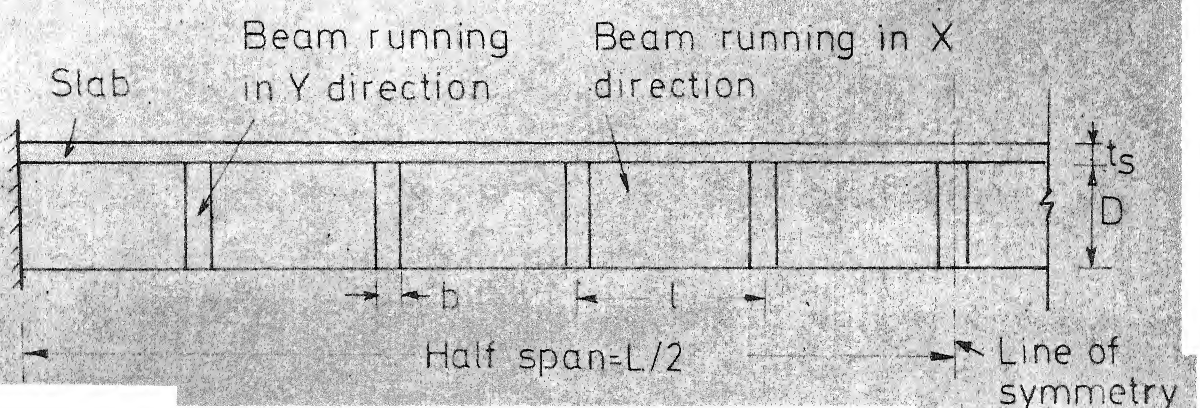
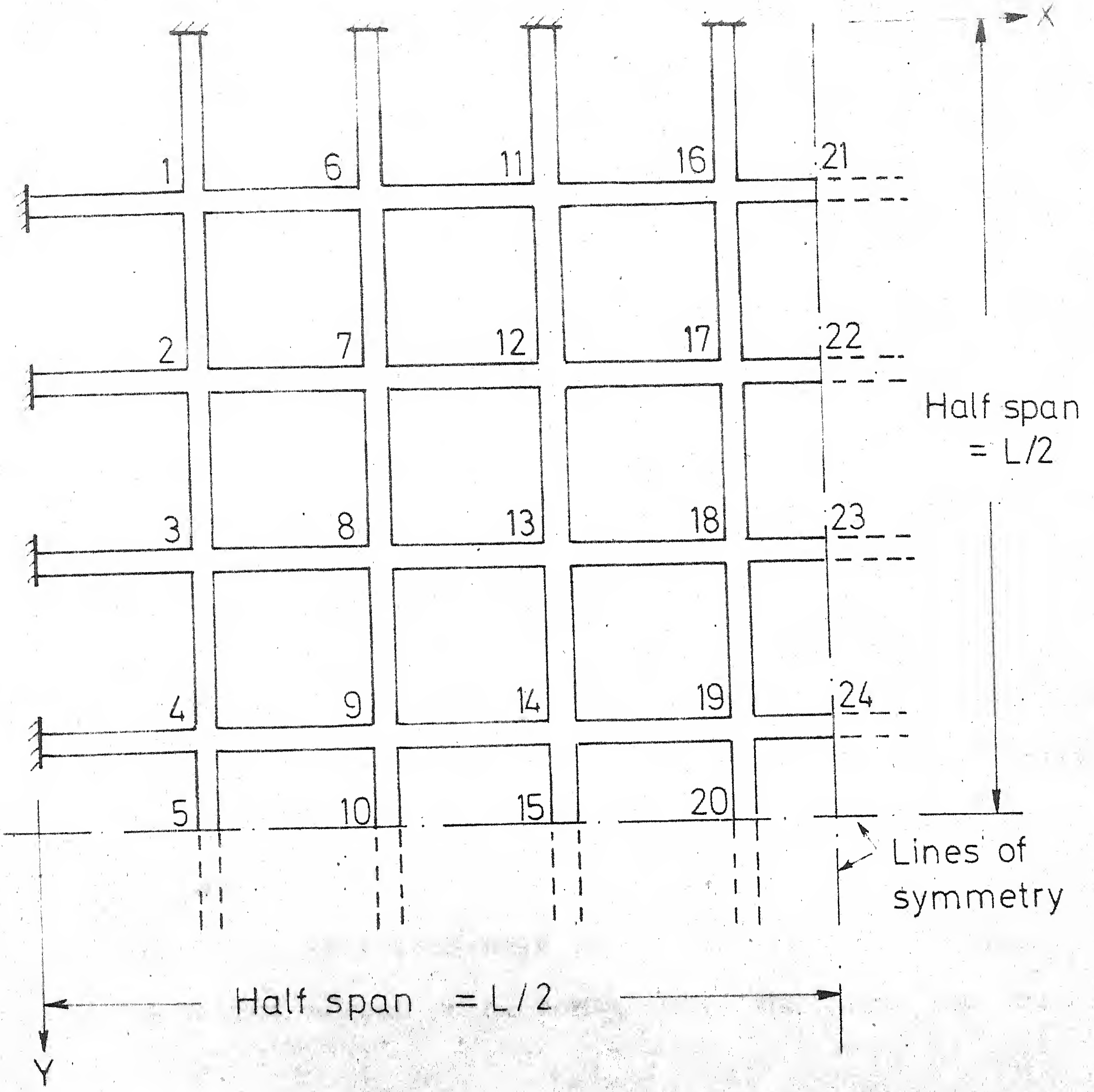
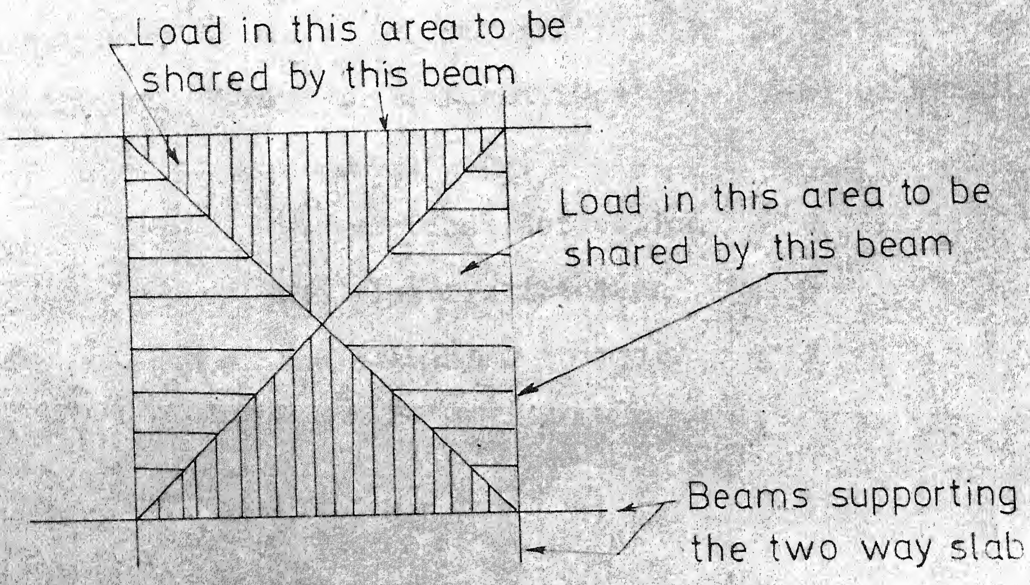


FIG. 2.2 SECTION OF GRID



FIG·2·3 GRID WITH EVEN NUMBER OF BEAMS IN EACH DIRECTION



FIG·2·4 DISTRIBUTION OF LOAD ON BEAMS

half the length of elements crossing the line of symmetry comes into picture. Here the points where the elements are cut are taken as nodes and the stiffnesses of these elements are calculated according to their length effective in the quarter.

2.4 LOAD ON BEAMS

As the spacing of the beams is same in both the directions the amount of load transferred by the slab, which acts as a two way continuous slab, to each element is same (see Figure 2.4). This load consists of live load on the slab and the dead weight of the slab and is taken to be uniformly distributed over the elements. This load plus the self weight of the beams constitutes the total load on beams.

2.5 FORCE DEFORMATION RELATIONS

The grid is subjected to transverse load only and as previously stated axial deformations of the elements are neglected. Hence the deformations which need be considered at each node of the element are,

the vertical deflection $= u$

the flexural rotation $= \theta$

and the torsional rotation $= \phi$

Corresponding forces at each node of the element are,

the shear force = V

the bending moment = M

and the torsional moment = T

Element notations for the forces and deformations of near and far end of the elements are shown in Figure 2.5. The superscripts N and F denote the near and far end quantities respectively. The near and far ends of the elements running in X and Y directions are shown in Figure 2.6. The global notations for forces and deformations are same as the element notations for elements running in X-direction.

Let us denote the nodes and elements as follows:

(i,j) is the node on i^{th} row and j^{th} column of the nodes.

x(i,j) is the element running in X-direction and having it's near end at node (i,j).

y(i,j) is the element running in Y-direction and having it's near end at node (i,j).

where $i = 1, 2, 3, \ldots$

$j = 1, 2, 3, \ldots$

Now the final forces at each node of elements x(i,j) and y(i,j) can be written as (refer Figure 2.6).

$$f^{N}_{x(i,j)} = f^{fN}_{x(i,j)} + k^{N}_{x(i,j)}\,U_{(i,j)} + ck^{F}_{x(i,j)}\,U_{(i,j+1)} \tag{2.1}$$

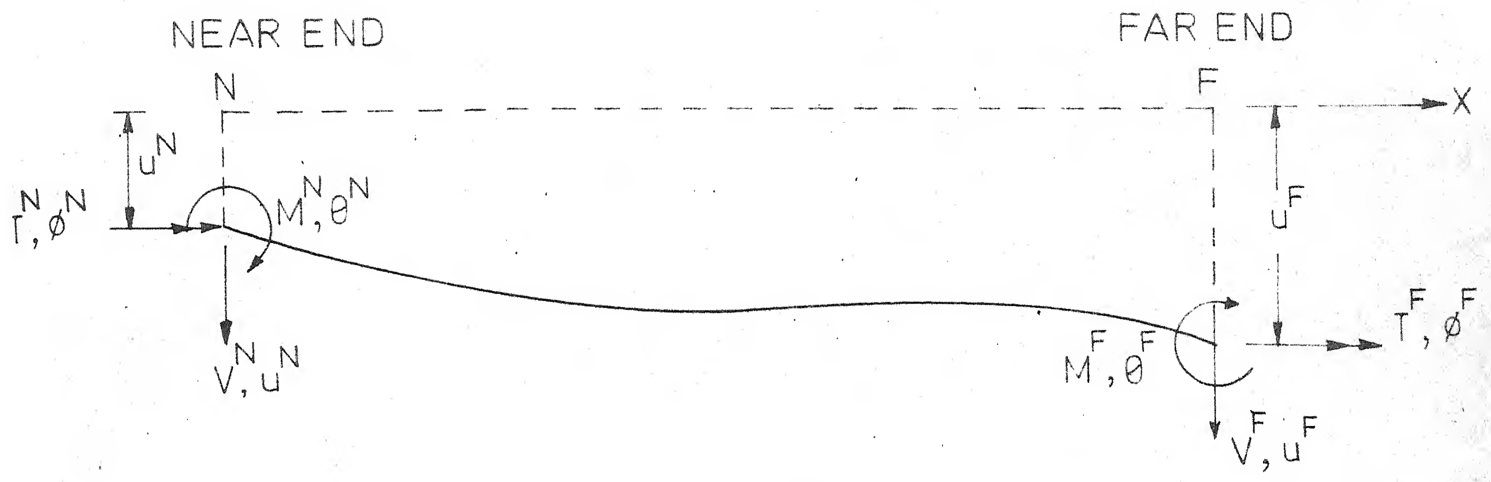


FIG-2·5 ELEMENT AND GLOBAL NOTATIONS FOR FORCES AND DEFORMATIONS

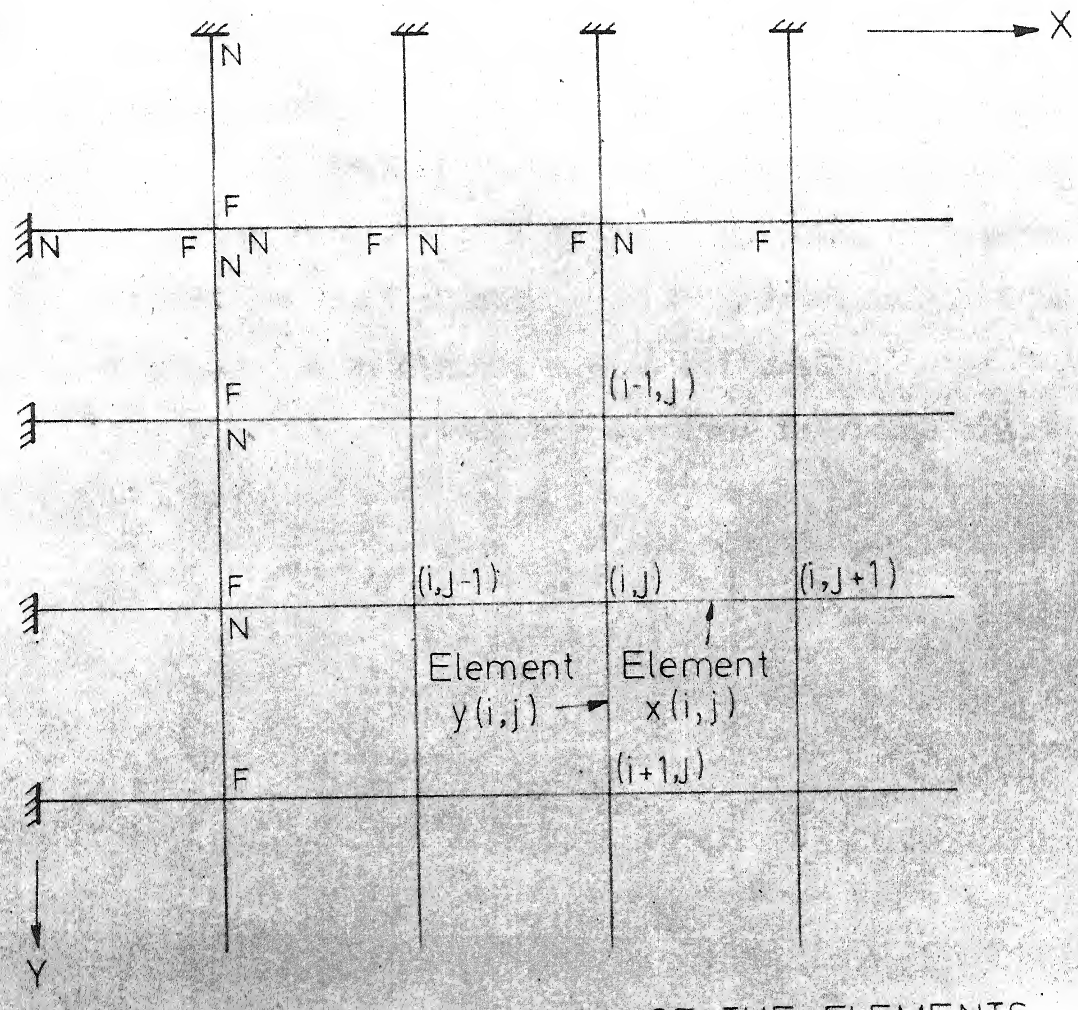


FIG. 2·6 NEAR AND FAR ENDS OF THE ELEMENTS

$$f^F_{x(i,j)} = f^{fF}_{x(i,j)} + ck^N_{x(i,j)}\, U_{(i,j)} + k^F_{x(i,j)}\, U_{(i,j+1)} \tag{2.2}$$

$$f^N_{y(i,j)} = t^* f^{fN}_{y(i,j)} + t^* k^N_{y(i,j)}\, t\, U_{(i,j)} + t^* ck^F_{y(i,j)}\, t\, U_{(i+1,j)} \tag{2.3}$$

and

$$f^F_{y(i,j)} = t^* f^{fF}_{y(i,j)} + t^* ck^N_{y(i,j)}\, t\, U_{(i,j)} + t^* k^F_{y(i,j)}\, t\, U_{(i+1,j)} \tag{2.4}$$

In the above equations the suffix $x(i,j)$, $y(i,j)$ denote the quantities corresponding to these elements. The suffix (i,j) denotes quantities corresponding to node (i,j). The superscripts N and F denote quantities corresponding to near and far ends of the elements respectively. Other notations for an element are as follows.

f is the final force vector in global notations given by

$$f = \begin{bmatrix} M \\ V \\ T \end{bmatrix}$$

f^f is the fixed end force vector in element notations.

k is the direct stiffness matrix of the element. For the near and far nodes the direct stiffness matrices are given by

$$k^N \equiv \begin{bmatrix} \frac{4EI}{l} & \frac{6EI}{l^2} & 0 \\ \frac{6EI}{l^2} & \frac{12EI}{l^3} & 0 \\ 0 & 0 & \frac{GJ}{l} \end{bmatrix} \quad \text{and} \quad k^F \equiv \begin{bmatrix} \frac{4EI}{l} & -\frac{6EI}{l^2} & 0 \\ -\frac{6EI}{l^2} & \frac{12EI}{l^3} & 0 \\ 0 & 0 & \frac{GJ}{l} \end{bmatrix}$$

Here

$I = \frac{bD^3}{12}$ is the moment of inertia of the element section

b is the width of the section

D is the depth of the section

l is the length of the element i.e. the spacing of beams

$GJ = G\beta b^3 D$ is the torsional rigidity of the section

E is the Young's modulus of elasticity for concrete

G is the modulus of rigidity for concrete

ck is the carryover stiffness matrix of the element. The carryover stiffness matrices for near and far node deformations are given by

$$ck^{N} \equiv \begin{bmatrix} \frac{2EI}{l} & \frac{6EI}{l^2} & 0 \\ -\frac{6EI}{l^2} & -\frac{12EI}{l^3} & 0 \\ 0 & 0 & -\frac{GJ}{l} \end{bmatrix}$$

and

$$ck^{F} \equiv \begin{bmatrix} \frac{2EI}{l} & -\frac{6EI}{l^2} & 0 \\ \frac{6EI}{l^2} & -\frac{12EI}{l^3} & 0 \\ 0 & 0 & -\frac{GJ}{l} \end{bmatrix}$$

U is the deformation vector in global notations given by

$$U \equiv \begin{bmatrix} \theta \\ u \\ \phi \end{bmatrix}$$

t is the transformation matrix, which by premultiplying to a deformation vector $U_{(i,j)}$ converts it into element notations for elements y(i,j).

$t^{*} = (t)^{T}$ = transformation matrix which by premultiplying to a force vector of element y(i,j) in element notations, converts it into global notations.

t and t^* are given by

$$t \equiv \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \qquad t^* \equiv \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

2.6 EQUATIONS OF EQUILIBRIUM

The equations of equilibrium of the forces at the node (i,j) can be written as

$$f^F_{x(i,j-1)} + f^N_{x(i,j)} + f^F_{y(i-1,j)} + f^N_{y(i,j)} = W_{(i,j)} \tag{2.5}$$

where $W_{(i,j)}$ is the external load vector at node (i,j) in global notations.

Using equations (2.1) through (2.4), equation (2.5) can be written as

$$\begin{aligned} & ck^N_{x(i,j-1)}\, U_{(i,j-1)} + t^* ck^N_{y(i-1,j)}\, t\, U_{(i-1,j)} \\ & + \left[k^F_{x(i,j-1)} + t^* k^F_{y(i-1,j)}\, t + t^* k^N_{y(i,j)}\, t + k^N_{x(i,j)} \right] U_{(i,j)} \\ & + t^* ck^F_{y(i,j)}\, t\, U_{(i+1,j)} + ck^F_{x(i,j)}\, U_{(i,j+1)} \\ & = W_{(i,j)} - f^{fF}_{x(i,j-1)} - t^* f^{fF}_{y(i-1,j)} - t^* f^{fN}_{y(i,j)} - f^{fN}_{x(i,j)} \end{aligned} \tag{2.6}$$

again this is written as

$$ck^{N}_{x(i,j-1)}\ U_{(i,j-1)} + CK^{N}_{y(i-1,j)}\ U_{(i-1,j)} + K_{(i,j)}\ U_{(i,j)}$$

$$+ CK^{F}_{y(i,j)}\ U_{(i+1,j)} + ck^{F}_{x(i,j)}\ U_{(i,j+1)}$$

$$= \overline{W}_{(i,j)} \tag{2.7}$$

where,

modified carryover stiffness matrix of element y(i-1,j) for it's near end deformations is given by

$$CK^{N}_{y(i-1,j)} = t^{*}\, ck^{N}_{y(i-1,j)}\ t$$

modified carryover stiffness matrix of element y(i,j) for it's far end deformations is given by

$$CK^{F}_{y(i,j)} = t^{*}\, ck^{F}_{y(i,j)}\ t$$

combined direct stiffness matrix at node (i,j) is given by

$$K_{(i,j)} = k^{F}_{x(i,j-1)} + t^{*}\, k^{F}_{y(i-1,j)}\ t + t^{*}\, k^{N}_{y(i,j)}\ t + k^{N}_{x(i,j)}$$

modified external load vector at node (i,j) is given by

$$\overline{W}_{(i,j)} = W_{(i,j)} - f^{fF}_{x(i,j-1)} - t^{*}\, f^{fF}_{y(i-1,j)} - t^{*}\, f^{fN}_{y(i,j)}$$

$$- f^{fN}_{x(i,j)}$$

Equation (2.7) is written for each node and all the equations are combined and written in the form

$$A\bar{X} = \bar{B} \tag{2.8}$$

where A = the stiffness matrix or coefficient matrix of the system

$\bar{X}$ = the deformation vector of the system

$\bar{B}$ = the modified external load vector of the system.

Equation (2.8) is the set of simultaneous equations with the deformation vector $\bar{X}$ as unknown.

2.7 SOLUTION OF THE SIMULTANEOUS EQUATIONS

The actual numbering of nodes is done as shown in Figures 2.1 and 2.3. This pattern gives a banded stiffness matrix of the system. The structure of the coefficient matrix A as given by equation (2.8) is shown in Figure 2.7. The solution of the equation (2.8) is obtained by Choleski's decomposition which is explained below.

The matrix A is symmetric and can be written as

$$A = S^T S \tag{2.9}$$

where S is an upper triangular matrix.

Using equation (2.9) in equation (2.8) we get

$$S^T S\bar{X} = \bar{B} \tag{2.10}$$

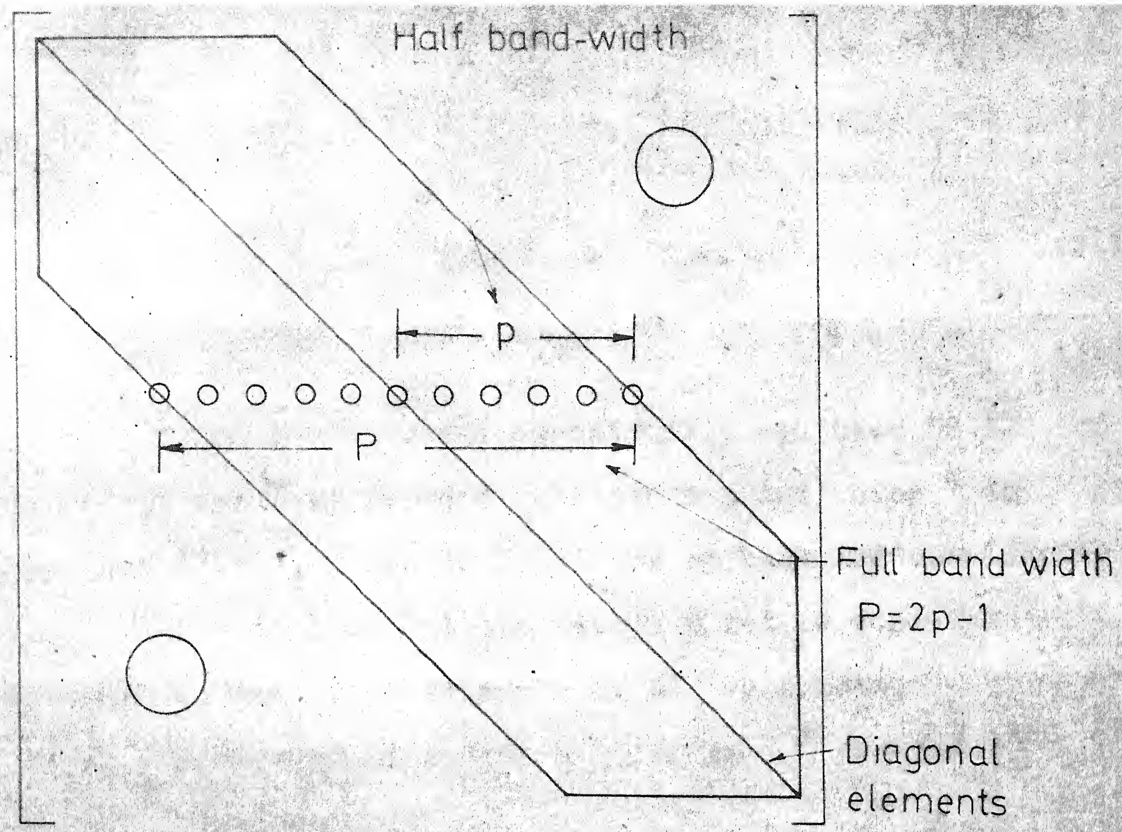


FIG. 2·7 STRUCTURE OF THE COEFFICIENT MATRIX 'A'

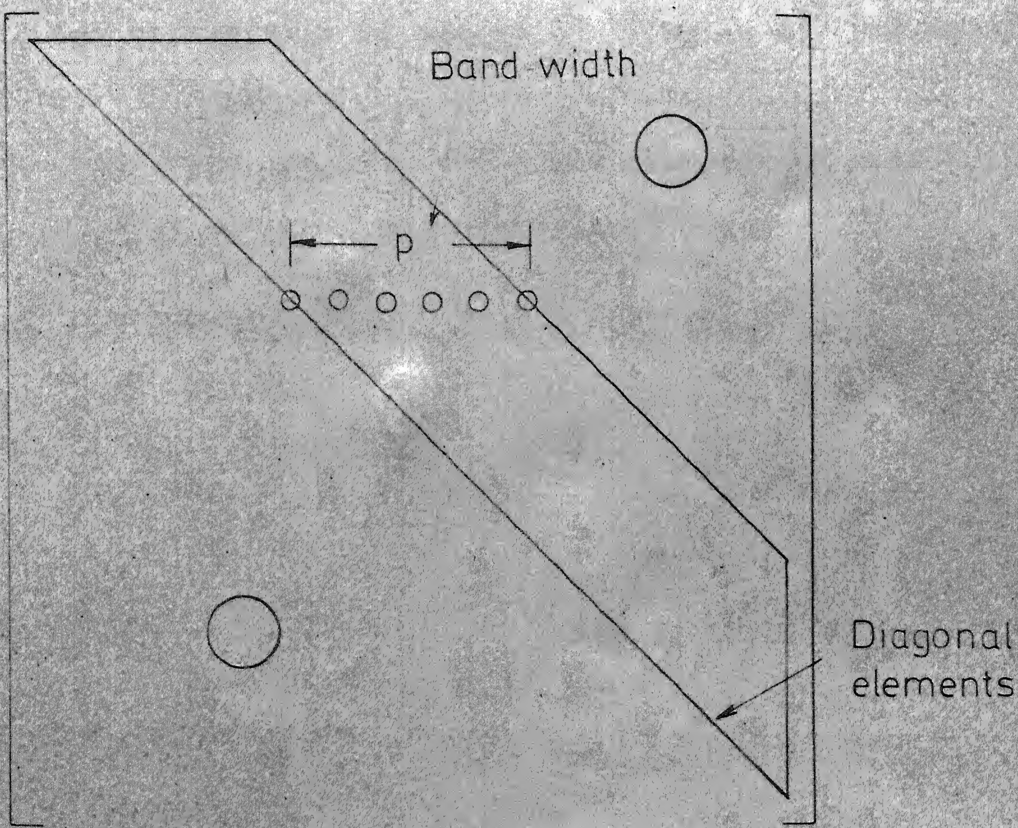


FIG. 2·8 STRUCTURE OF THE DECOMPOSED MATRIX 'S'

putting $S\bar{X} = \bar{Y}$ in above equation we get, (2.11)

$$S^T\bar{Y} = \bar{B} \tag{2.12}$$

Once the matrix S is obtained, equation (2.12) can be solved for $\bar{Y}$ by forward substitution and using $\bar{Y}$ in equation (2.11), $\bar{X}$ can be found out by backward substitution.

Now to find out the matrix S let us consider the elements in the upper triangle of A. We denote,

$a_{i,j}$ = element of matrix A lying in i^{th} row and j^{th} column

$s_{i,j}$ = element of matrix S lying in i^{th} row and j^{th} column

$s'_{i,j}$ = element of matrix S^T lying in i^{th} row and j^{th} column.

From equation (2.9) the elements of matrix A can be written in terms of the elements of S as,

$$a_{i,j} = s'_{i,1}s_{1,j} + s'_{i,2}s_{2,j} + \dots + s'_{i,i}s_{i,j} \qquad i \leq j \tag{2.13}$$

$$= s_{1,i}s_{1,j} + s_{2,i}s_{2,j} + \dots + s_{i,i}s_{i,j} \qquad i \leq j \tag{2.14}$$

$$(\text{as } s'_{i,j} = s_{j,i})$$

For diagonal elements putting $j = i$ in equation (2.14) we get,

$$a_{i,i} = s_{1,i}^2 + s_{2,i}^2 + \ldots + s_{i,i}^2 \qquad (2.15)$$

From equations (2.14) and (2.15)

$$s_{i,j} = \frac{a_{i,j} - s_{1,i}s_{1,j} - s_{2,i}s_{2,j} - \ldots - s_{i-1,i}s_{i-1,j}}{s_{i,i}} \qquad (2.16)$$

and $$s_{i,i} = (a_{i,i} - s_{1,i}^2 - s_{2,i}^2 - \ldots - s_{i-1,i})^{1/2} \qquad (2.17)$$

Using equations (2.16) and (2.17) elements of S can be found out.

The structure of the matrix S is also banded with band width equal to half the band width of matrix A. This can be easily explained as follows. Consider an off-band element $a_{1,j}$ of matrix A in the first row

$$\therefore \quad a_{1,j} = 0 \qquad j > p \qquad (2.18)$$

from equation (2.16) we have,

$$s_{1,j} = \frac{a_{1,j}}{s_{1,1}} \qquad j > p$$

$$= 0 \qquad \text{from equation (2.18).}$$

Extending the argument for elements in other rows it can be proved that

$$s_{i,j} = 0 \qquad\qquad j - i \geq p$$

The structure of matrix S is shown in Figure 2.8.

2.8 FORCES IN THE ELEMENTS

After the equation (2.8) are solved for deformations the final forces at each node of the elements can be found out with the help of equations (2.1) to (2.4).

CHAPTER 3

DESIGN OF SECTIONS

3.1 INTRODUCTION

Working stress design method is used for designing the sections of each element of the grid in the present work. Forces at both nodes and at the centre of the element being known from analysis, the maximum force is taken for designing the element section. The section is designed for three forces viz., the bending moment, the torsional moment and the shear force. Specifications from IS code of practice (18) are followed for the design.

3.2 ASSUMPTIONS

Following assumptions are made in designing the section.

1) Plane cross-sections before bending remain plane after bending.

2) For concrete stress is directly proportional to strain i.e. Young's modulus of elasticity is constant at all stresses.

3) Concrete area on tension side is ineffective.

4) Reinforcing steel is concentrated at the centroid of the steel area.

3.3 DESIGN OF SLAB

The slab runs over a number of beams running at right angles to each other. Hence it acts as a two way continuous slab and is designed accordingly. The design is done according to "The method 3" as described in IS-456-1964. In this method tables are presented which give coefficients for maximum bending moment in a two way continuous slab with given aspect ratio.

Minimum thickness of slab is taken as larger of the following:

1) $t_{smin} = 10$ cms

2) $t_{smin} = \frac{1}{40}$

where t_{smin} is minimum thickness of the slab.

It is seen that for a given spacing, specified minimum thickness of slab is sufficient to carry the load safely. This is true at all the spacings considered in the present work.

Minimum percentage of reinforcement in the slab is taken to be equal to 0.15% of the cross-section.

3.4 DESIGN OF BEAM FOR FLEXURE

The beam is designed for flexure such that the section is either singly or doubly reinforced. When the bending moment at the section is less than or equal to the moment of resistance of the balanced section (MRb) the section is designed as a singly reinforced beam and in other case i.e. when the bending moment is greater than MRb the section is designed as a doubly reinforced beam. Following sections explain briefly the design of singly and doubly reinforced beams.

3.4.1 Design of a Singly Reinforced Section

Referring to Figure 3.1 the position of neutral axis and moment of resistance of a singly reinforced section can be obtained from equations

$$\frac{bn^2}{2} = mA_s(d - n) \tag{3.1}$$

$$MR = \sigma_s' A_s(d - \frac{n}{3}) \tag{3.2}$$

where

n is the distance of neutral axis from the extreme fibre in compression

d is the effective depth of beam

m is the modular ratio for steel and concrete

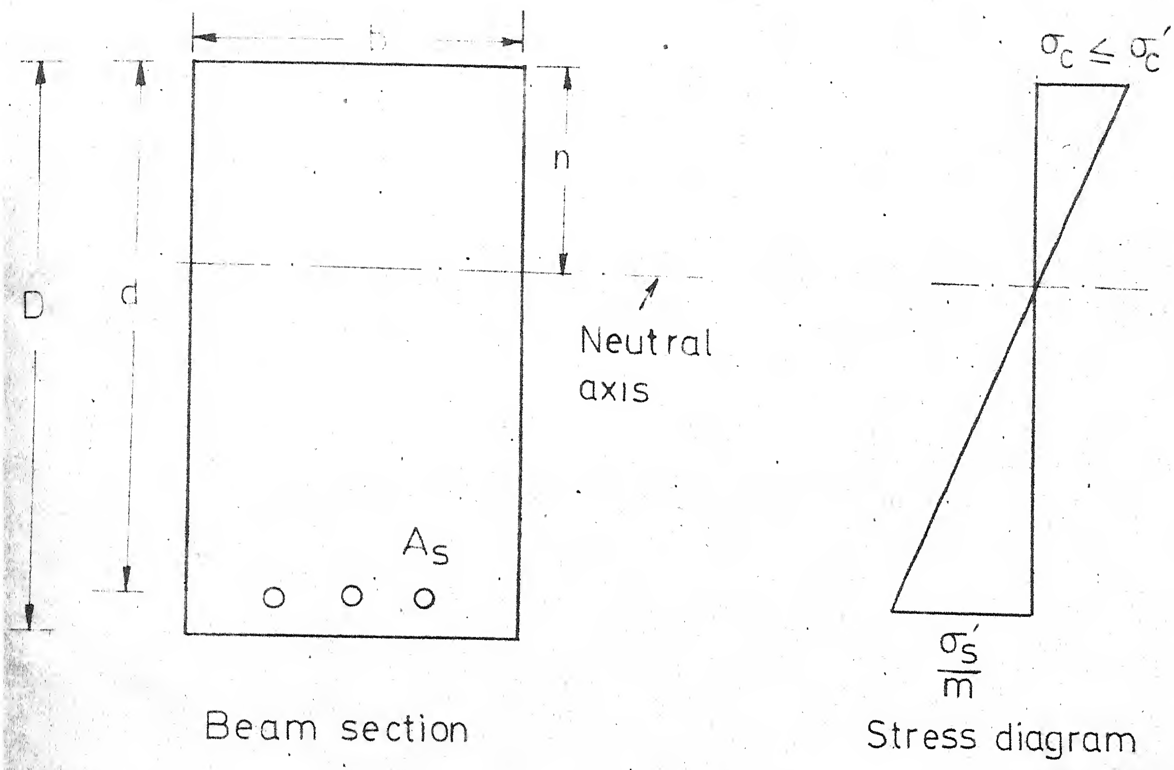


FIG. 3.1 SINGLY REINFORCED SECTION

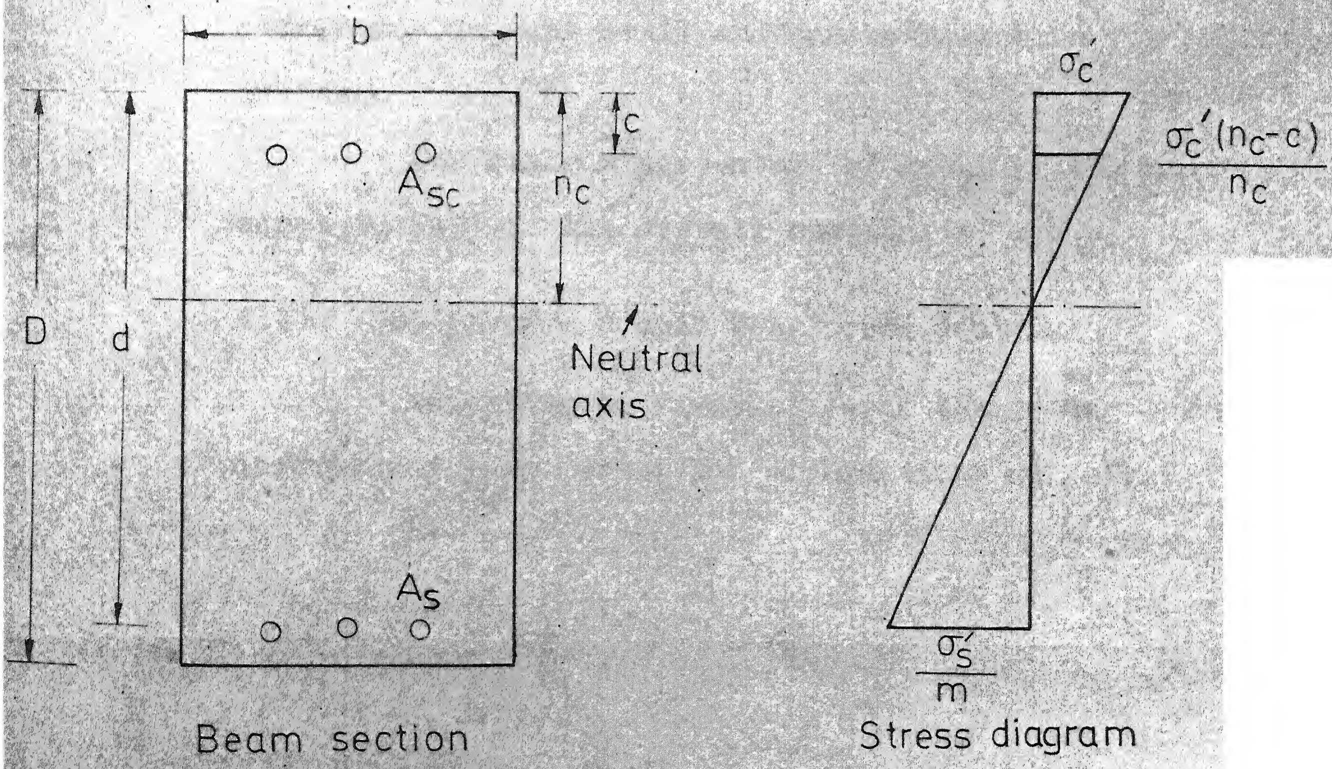


FIG. 3.2 DOUBLY REINFORCED SECTION

MR is the moment of resistance of the section

A_s is the area of steel in tension

and σ'_s is the maximum permissible stress in steel.

To find the steel area A_s such that MR is equal to the external bending moment M, is a trial and error process. These trials are eliminated in the present work with the following approximation.

$$A_s = \frac{A_{sb} \times M}{MRb}$$

where

A_{sb} is the area of tension steel for the balanced section.

The steel area obtained from the above approximation is slightly more than the actual required steel. But the difference is very small and can be neglected for practical purposes.

The minimum percentage of steel in the beam is kept equal to 0.3% of the overall section.

3.4.2 Design of a Doubly Reinforced Section

Referring to Figure 3.2 and taking the moment of compression force about the neutral axis we get,

$$MR = bn_c \frac{\sigma_c'}{2}\left(d - \frac{n_c}{3}\right) + (m - 1)A_{sc}\,\sigma_c'\left(\frac{n_c - c}{n_c}\right)(d - c) \tag{3.1}$$

$$= MRb + M_1$$

where

$$MRb = bn_c \frac{\sigma_c'}{2}\left(d - \frac{n_c}{3}\right) \tag{3.2}$$

is the moment of resistance due to concrete area,

$$M_1 = (m - 1)A_{sc}\,\sigma_c'\left(\frac{n_c - c}{n_c}\right)(d - c) \tag{3.3}$$

is the moment of resistance due to compression steel,

n_c is the distance of the neutral axis from the extreme compression fibre in a balanced section,

σ_c' is the maximum permissible bending compressive stress in concrete,

A_{sc} is the area of compression steel,

and c is the cover for compression steel.

Equating MR and M we have

$$MR = M = MRb + M_1 \tag{3.4}$$

$$\therefore \quad M_1 = BM - MRb \tag{3.5}$$

MRb can be computed for a given section by using equation (3.2). Furthermore, using equations (3.3) and (3.5) A_{sc} can be found out.

Now equating moment of areas about the neutral axis we get,

$$mA_s(d - n_c) = (m - 1)A_{sc}(n_c - c) + b\,n\frac{n_c}{2} \qquad (3.6)$$

where from the only unknown A_s i.e. the tension steel area can be computed.

The value of A_{sc} is subject to a maximum of 4% of the gross section as per IS code of practice.

3.5 DESIGN FOR SHEAR AND TORSION

The shear stresses in a section due to the shear force and torsional moment are given by

$$q_1 = \frac{V}{b\,a}$$

and
$$q_2 = \frac{T(3 + \frac{2b}{D})}{b^2 D}$$

where

q_1 is the shear stress due to shear force V

q_2 is the shear stress due to torsional moment T

a is the lever arm of the section.

Now, if $q_1 + q_2 < q'$ the maximum permissible shear stress in concrete, nominal shear steel is provided.

if $q_1 + q_2 > 4(q')$ the section is discarded and if $q' < q_1 + q_2 < 4(q')$ shear steel is designed as follows.

For shear force

$$A_{ss1} = \frac{q_1 b}{\sigma'_{ss}}$$

and for torsional shear

$$A_{ss2} = \frac{T}{0.8\ \sigma'_s X_1 Y_1}$$

where A_{ss1} is the steel area per unit length in all legs of stirrups for shear force

A_{ss2} is the steel area per unit length in all legs of stirrups for torsional shear

σ'_{ss} is the permissible tension in shear steel

and X_1 and Y_1 are the sides of the rectangular stirrups.

In addition to stirrups longitudinal steel is provided such that it has the same volume per unit length as the volume contained in all legs of stirrups resisting torsional shear.

CHAPTER 4

FORMULATION OF THE MATHEMATICAL PROGRAMMING PROBLEM

4.1 INTRODUCTION

Once the method of analysis and the design philosophy are decided upon, the optimum design problem can be cast as a mathematical programming problem which has the following form.

$$\text{Minimize } F(\bar{D}) \;, \quad \bar{D} = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_N \end{bmatrix}$$

Subject to conditions

$$g_i(\bar{D}) \leq 0 \qquad i = 1, 2, \ldots, NC$$

where

$F(\bar{D})$ is the objective function

N is the number of design variables

$g_i(\bar{D})$ is the i^{th} constraint

and NC is the total number of constraints.

The details regarding the design variables, objective function, constraints and the technique for optimization are explained in the following sections.

4.2 THE DESIGN VARIABLES

Design variables in an optimization problem are the independent quantities for which values are to be assigned in producing a design. For designing a grid roof of given size and with a given number of beams the quantities which are required to be considered independently are, the slab thickness, width of the beams and depth of the beams. In the present work the slab thickness is always kept to minimum as required by code in order to keep the dead weight on the grid at it's minimum. Hence only two quantities viz. width of the beams and depth of the beams form the design variable vector.

4.3 THE OBJECTIVE FUNCTION

For a grid with given number of beams i.e. for a given spacing of beams the cost of slab is constant as it's thickness is kept constant. Hence only cost of beams is going to change as the design variables change. So the objective function considered for optimization is the cost of beams, the expression for which is as follows:

$$F(b,D) = Nb(Nb + 1)\, b\, D(2l - b)C_c$$

$$+ C_s l \sum_{i=1}^{NE} \left[A_{si} + A_{sci} + (A_{ss1i} + 2A_{ss2i})(X_1 + Y_1) \right]$$

where F is the cost of beams in the grid

C_c is the cost of concrete per unit volume

C_s is the cost of steel per unit volume

NE is the total number of elements in the grid

$= Nb(Nb + 1)2.$

The cost of materials are taken from Delhi Schedule of Rates (2). Cost of concrete is taken as Rs. 166.6 per m^3. It does not include the cost of formwork. The cost of reinforcement is taken as Rs. 2.2 per Kg which includes the cost of bending, binding and placing in position.

4.4 THE CONSTRAINTS

Constraints in working out a design are the conditions which when violated give an infeasible design. In a design problem certain limitations are imposed on the behaviour of the structure to get a safe design. These limitations are called as the behaviour constraints. Usually there are still other limitations required to be considered

so as to obtain a practicable design. These are termed as side constraints. For the present work following constraints are considered to get a safe and practicable optimal design.

The behaviour constraints are:

(1) limitation on maximum shear stress

$$g_{1(j)} = q_{1(j)} + q_{2(j)} - 20 \leq 0 \qquad (4.1)$$

$$j = 1, 2, \ldots, NED$$

(2) limitation on the maximum compression steel area

$$g_{2(j)} = A_{sc(j)} - \frac{b\ D}{25} \leq 0 \qquad (4.2)$$

$$j = 1, 2, \ldots, NED$$

where NED is the number of elements designed.

The side constraints are as follows:

(3) limitation on the minimum width of beam. This is taken as the critical of the following:

a) limitation on the absolute minimum width of beam

$$g_3 = 15 - b \leq 0 \qquad (4.3)$$

b) limitation on the width of beam so as to avoid a slender beam

$$g_3 = 1 - 30b \leq 0 \qquad (4.4)$$

(4) limitation on the maximum width of beam

$$g_4 = b - \frac{1}{2} \leq 0 \qquad (4.5)$$

(5) limitation on the non-negativity of the depth

$$g_5 = -D \leq 0 \tag{4.6}$$

4.5 TECHNIQUE FOR OPTIMIZATION

The constrained minimization problem is transformed into a sequential unconstrained minimization problem with the use of interior penalty function. The penalty function used for present work is

$$\phi(\bar{D}, r) = F(\bar{D}) - r \sum_{i=1}^{NC} \frac{1}{g_i(\bar{D})} \tag{4.7}$$

where $\phi(\bar{D}, r)$ is the penalty function

and r is the penalty parameter.

Sequential unconstrained minimization comprises of the following steps:

1) With a feasible design point $\bar{D}_o$ and an initial value of r minimize $\phi(\bar{D}, r)$ to obtain $\bar{D}_m$.

2) Check the convergence for $\bar{D}_m$ to be optimum design. If satisfied terminate otherwise go to next step.

3) Reduce the value of r by $r \leftarrow hr$ where h is the reduction factor.

4) Compute a new starting point and repeat the minimization process from step 1.

To obtain a feasible starting point in order to initiate the optimization algorithm, only a few trials were required in the present problem. Initial value of r was chosen such that at the starting point $\overline{D}_o$ the penalty imposed is equal to the objective function value. In the sequential process of minimization r was reduced by the factor h = 0.1 till it reached a value equal to 0.01 and then the algorithm was terminated. The starting point chosen for the reduced r was the minimum point obtained for the previous value of r.

The unconstrained minimization of $\phi(\overline{D}, r)$ was carried out by the variable metric (DFP) method. Brief outline of this method is presented below.

1) Set q = 0.

2) Start with an initial design point $\overline{D}_o$ and an initial positive definite matrix H_o which is taken as identity matrix in the present work and set the initial direction vector

$$\overline{S}_o = - H_o \nabla \phi(\overline{D}_q, r)$$

3) Compute $\overline{D}_{q+1} = \overline{D}_q + \alpha_q^* \overline{S}_q$

where α_q^* is the minimizing step length in the direction $\overline{S}_q$ giving $\overline{D}_{q+1}$ as the point at minimum.

4) Check the convergence for $\overline{D}_{q+1}$ to be the optimum design point. If satisfied terminate otherwise go to next step.

5) Compute

$$H_{q+1} = H_q + \alpha_q^* \frac{\overline{S}_q \overline{S}_q^T}{\overline{S}_q^T \overline{Z}_q} - \frac{(H_q \overline{Z}_q)(H_q \overline{Z}_q)^T}{\overline{Z}_q^T H_q \overline{Z}_q}$$

where $\overline{Z}_q = \nabla \phi(\overline{D}_{q+1}, r) - \nabla \phi(\overline{D}_q, r)$

6) Compute $\overline{S}_{q+1} = - H_{q+1} \quad \phi(\overline{D}_{q+1}, r)$

7) Set $q \leftarrow q+1$ and repeat from step 3.

The convergence criterion used in step 4 is that the percentage reduction in the penalty function value during two successive linear minimizations is less than 1%. The gradient evaluation is done by using the forward difference technique and for linear minimization quadratic interpolation is used. The quadratic was fitted to an accuracy of 1%.

CHAPTER 5

RESULTS, DISCUSSION AND CONCLUSIONS

5.1 GENERAL

The application of the formulation described in the previous chapter is explained with three illustrative examples in the following sections. The results presented in this chapter are obtained on IBM 7044/1401 system at Indian Institute of Technology, Kanpur.

The numerical values pertaining to material properties used for solving illustrative problems are as follows:

1)	Grade of concrete	M150
2)	Reinforcing steel	M S Grade I
3)	Density of reinforced cement concrete	2400 Kg/m^3
4)	Density of reinforcing steel	7880 Kg/m^3
5)	Maximum permissible bending compressive stress in concrete	50 Kg/cm^2
6)	Maximum permissible tensile stress in reinforcement	1400 Kg/cm^2
7)	Modular ratio for steel and concrete	18

5.2 ILLUSTRATIVE EXAMPLE 1

In this example a grid with overall size 18.82 M x 18.82 M (Grid 1) is considered to find the optimum spacing of beam leading to the minimum cost of the roof. For illustration let us consider a case when the number of beams running in each direction are even, say equal to 8. The quarter of this structure (Figure 2.3) has 24 nodes. Thus the total number of degrees of freedom turn out to be 72. The spacing of beams is 209.11 cms centre to centre. Therefore, the slab thickness comes out to be 10 cms i.e. the minimum to be provided. This gives a dead weight of 240 Kg/m^2 on the grid. In addition to this dead weight, a live load of 75 Kg/m^2 on the slab and the dead weight of the grid (which varies as the design changes) are also considered.

As indicated in Chapter 4, the optimum design problem is a two variable problem with the width and depth of beams as design variables. Therefore $\bar{D}^T \equiv (b, D)$.

The width of the beam is allowed to vary in the range of values such that the beam does not become a slender beam on one hand and the beams still maintain a spacing between them. In other words, for the example considered

$$6.97 = \frac{209.11}{30} \leq b \leq \frac{209.11}{2} = 104.5$$

In order to make the lower and the upper bounds worked out above as practicable the constraint actually imposed on the width is

$$15 \leq b \leq 60$$

The depth of the beam is only constrained to be non-negative.

The difference in the levels of tension reinforcement in X and Y direction beams is taken into account by providing a cover of 4 cms and 6.5 cms in the two directions respectively. The cover for compression reinforcement is assumed to be $1/10^{th}$ of the effective depth of the section.

The cross-section of the grid between two nodes is designed for maximum bending moment, shear force and torsion and reinforcing steel is accordingly provided. Hence for each design twenty cross-sections are to be detailed in each direction. For each cross-section the maximum compression steel and maximum shear stress are constrained. Thus the optimum design problem turns out to be having a total of 83 constraints.

The problem has been solved with a starting point (45.0, 130.0) having a total cost of Rs. 62,007. The proposed optimum design comes out as (15.02, 129.39) having a total cost of Rs. 26,457. The computer time taken for this problem is 349 secs.

A parametric study with different number of beams ranging from 2 in increments of one upto 10 beams in each direction has been made. The results of the parametric study are shown in Table 1. A glance on the table shows that optimum number of beams turns out to be 4 with a spacing of 376.4 cms. The total cost of the optimum design is Rs. 20,763. A plot of number of beams in each direction versus total cost of roof is shown in Figure 5.1.

As a check for the optimum design to be local or global minimum, minimization is carried out with one more starting point (35.0, 100.0) which gives the optimum cost of roof as Rs. 20,766 at a section (15.05, 124.37). Since in the two cases the optimum design reached is similar and so also the optimum cost , it can be said with some degree of confidence that the proposed optimum design is the global minimum of the problem.

5.3 ILLUSTRATIVE EXAMPLE 2

A second grid with overall size 30 M x 30 M (Grid 2) is optimized for the purpose of studying the optimum spacing of the beams. For illustration consider the grid when the number of beams are odd, say equal to 5. The quarter of this structure has 9 nodes resulting in 27 degrees of

Table 1

Grid 1 : Size 18.82 M x 18.82 M

No. of beams in each direction Nb	Starting design point			Proposed optimum design point		
	b cms	D cms	Cost of roof (Rs.)	b cms	D cms	Cost of roof (Rs.)
10	40.00	150.00	62,476	15.08	145.43	31,021
9	40.00	120.00	52,789	15.00	122.13	27,136
8	45.00	130.00	62,007	15.02	129.39	26,457
7	45.00	135.00	57,187	15.04	133.00	25,034
6	50.00	160.00	55,874	15.00	113.69	22,825
5	50.00	150.00	54,311	15.00	140.90	22,105
4*	55.00	150.00	52,178	15.08	130.85	20,763
	35.00	100.00	31,725	15.05	124.37	20,766
3	45.00	160.00	37,188	15.91	157.97	22,064
2	40.00	140.00	35,303	20.97	156.06	28,430

* Optimum number of beams: 4-4 with a spacing of 376.4 cms.

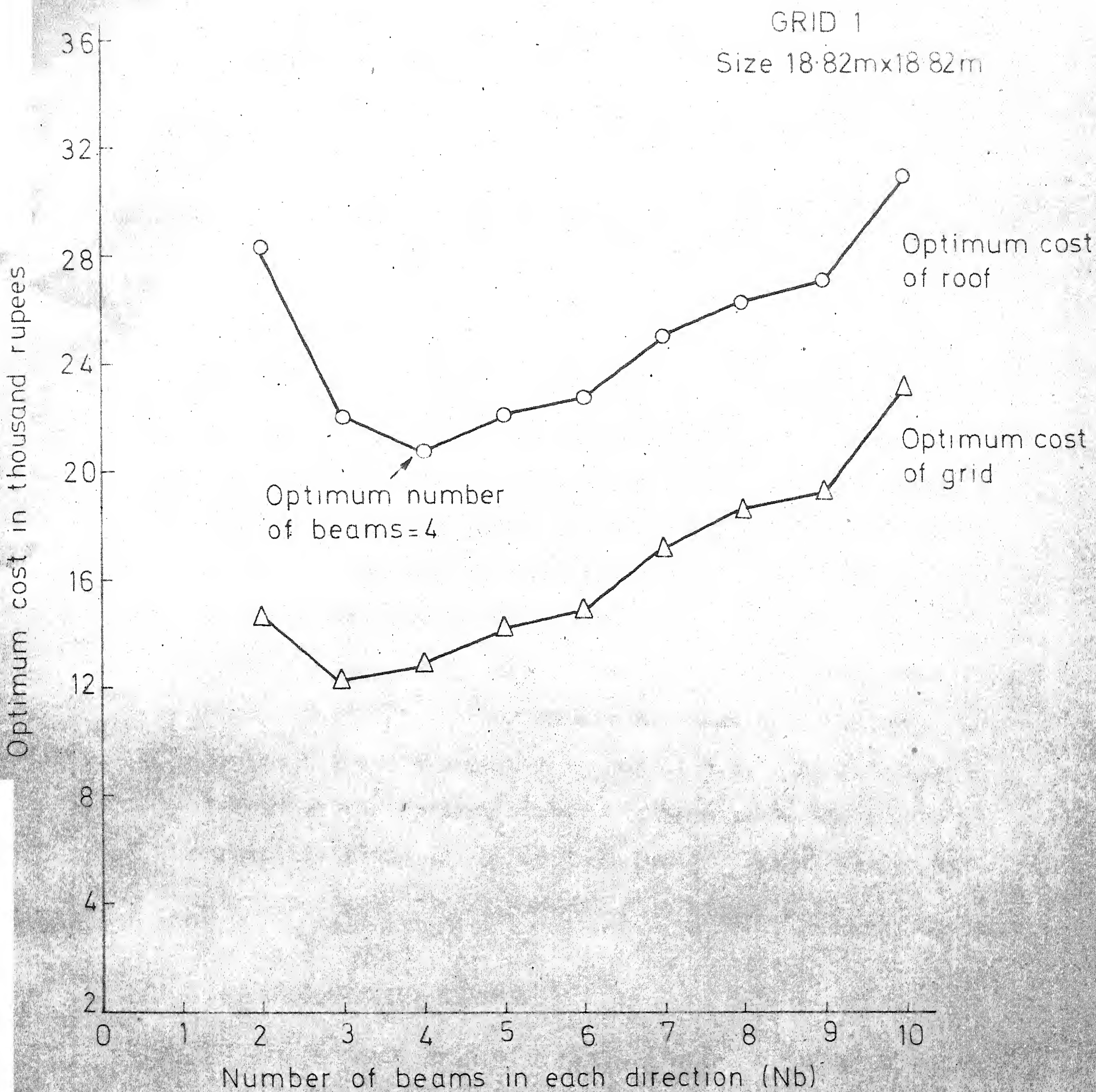


FIG. 5·1 VARIATION IN THE OPTIMUM COST OF GRID AND THE OPTIMUM COST OF ROOF WITH RESPECT TO THE CHANGE IN THE NUMBER OF BEAMS IN EACH DIRECTION FOR GRID 1

freedom. The spacing of beams is 500 cms. Hence the slab thickness turns out to be 12.5 cms, the minimum to be provided. Thus the dead weight on the grid is 300 Kg/m^2. All other details are worked out as explained in example 1. This problem turns out to have 2 design variables and 39 constraints.

Taking the starting point as (40.0, 140.0) having a total cost of Rs. 1,33,396 the proposed optimum design obtained is (16.67, 246.00) having a total cost of Rs. 79,745.

A parametric study with respect to number of beams is made in the range Nb equal to 5 to 12. Table 2 shows the results of this parametric study. It is seen from Table 2 that the optimum number of beams is 6 having a spacing between them equal to 444.44 cms. The total cost of roof at the optimum number of beams is Rs. 73,340. The parametric study of the optimal design taking Nb as a parameter is represented graphically in Figure 5.2.

5.4 ILLUSTRATIVE EXAMPLE 3

A third grid with overall size 40 M x 40 M (Grid 3) is studied for optimization with respect to the spacing of beams. This grid is optimized for the number of beams Nb in the range of 5 to 12. The details for each case are worked out as explained in previous examples. The results

Table 2

Grid 2 : Size 30 M x 30 M

No. of beams in each direction Nb	Starting design point			Proposed optimum design point		
	b cms	D cms	Cost of roof (Rs.)	b cms	D cms	Cost of roof (Rs.)
9	45.00	200.00	1,74,928	15.01	172.47	79,905
8	50.00	230.00	1,86,093	15.00	189.78	76,495
7	50.00	225.00	1,69,982	15.07	224.25	73,864
6*	50.00	225.00	1,59,510	15.04	225.12	73,346
5	40.00	140.00	1,33,396	16.67	246.80	79,745
4	55.00	260.00	1,64,356	20.00	259.00	90,961
3	55.00	260.00	1,56,864	25.12	320.56	1,07,068

* Optimum number of beams: 6-6 with a spacing of 428.57 cms.

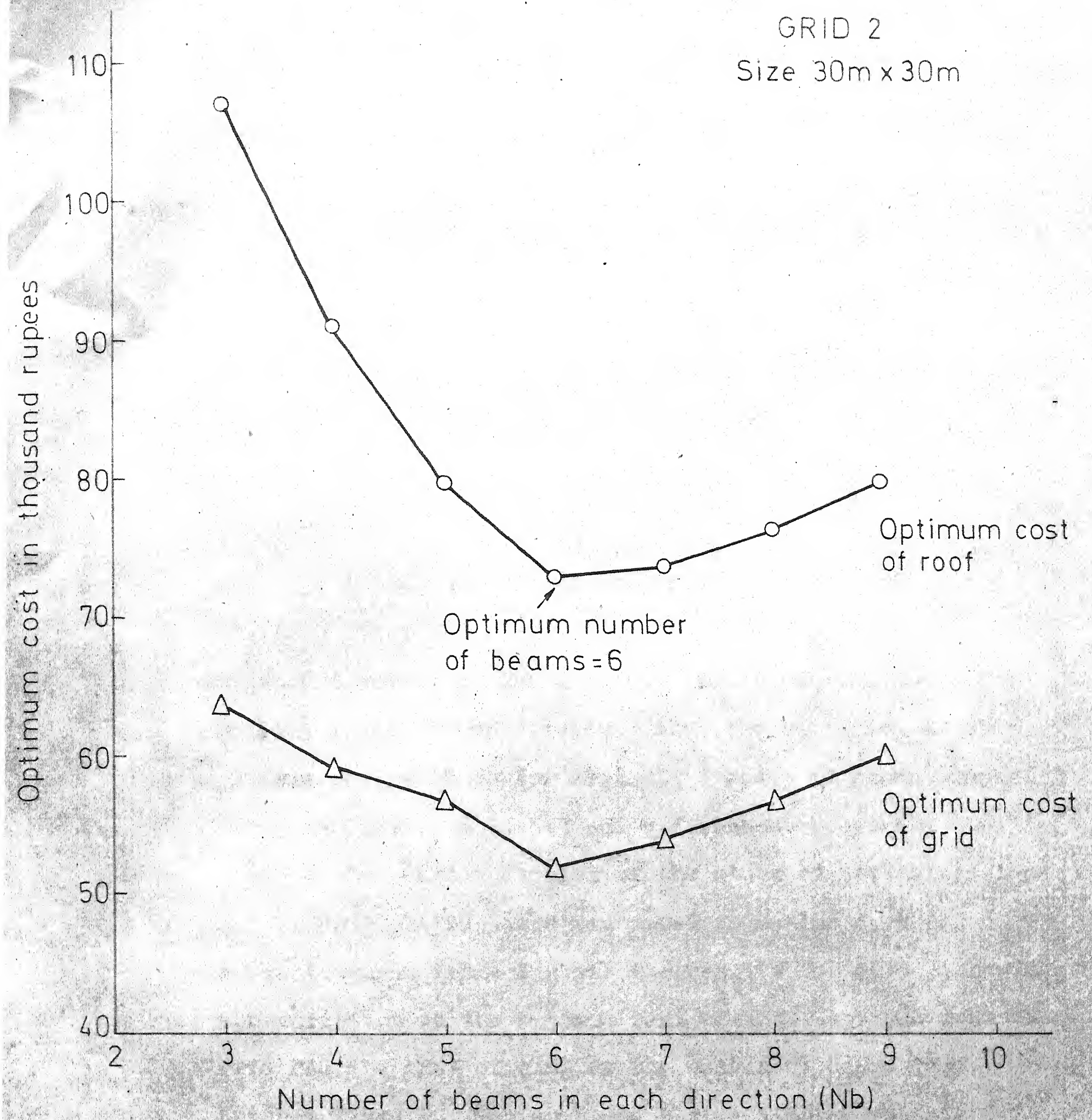


FIG. 5.2 VARIATION IN THE OPTIMUM COST OF ROOF AND THE OPTIMUM COST OF GRID WITH RESPECT TO THE CHANGE IN THE NUMBER OF BEAMS IN EACH DIRECTION FOR GRID 2

of the parametric study with respect to number of beams are given in Table 3. The variation of optimum cost with the number of beams in each direction is shown graphically in Figure 5.3.

From Table 3 it is seen that the optimum number of beams Nb leading to the minimum cost design is 8 at which the beams are at a spacing of 444.44 cms.

5.5 RESULTS

As already stated the total cost of roof at the starting point and at the imposed optimum design for different values of Nb are given in Tables 1, 2 and 3 for Grids 1, 2 and 3 respectively. Also the variation in the optimum cost with Nb for Grids 1, 2 and 3 is shown graphically in Figures 5.1, 5.2 and 5.3 respectively.

The detail results of the study of optimal designs for Grids 1, 2 and 3 are presented in Tables 4, 5 and 6 respectively. Table 4 gives the results for Grid 1 showing the variation in the optimum cost of roof, optimum cost of grid and the cost of slab as the number of beams vary between 2 and 9. Tables 5 and 6 give similar results for Grids 2 and 3 when Nb varies in the range of 3 to 9 and 5 to 12 respectively.

Table 3

Grid 3 : Size 40 M x 40 M

No. of beams in each direction Nb	Starting design point			Proposed optimum design point		
	b cms	D cms	Cost of roof (Rs.)	b cms	D cms	Cost of roof (Rs.)
12	50.00	320.00	5,00,142	15.06	327.55	1,95,348
11	45.00	190.00	3,91,109	15.06	320.49	1,85,512
10	50.00	310.00	4,42,023	15.01	318.75	1,76,179
9	55.00	360.00	4,99,841	15.09	336.00	1,73,294
8*	60.00	350.00	4,65,450	15.09	348.38	1,70,441
7	60.00	350.00	4,46,593	16.73	349.22	1,80,655
6	65.00	335.00	4,46,118	19.19	341.60	1,95,031
5	60.00	340.00	4,00,447	22.30	359.64	2,18,732

* Optimum number of beams: 8-8 with a spacing of 444.44 cms.

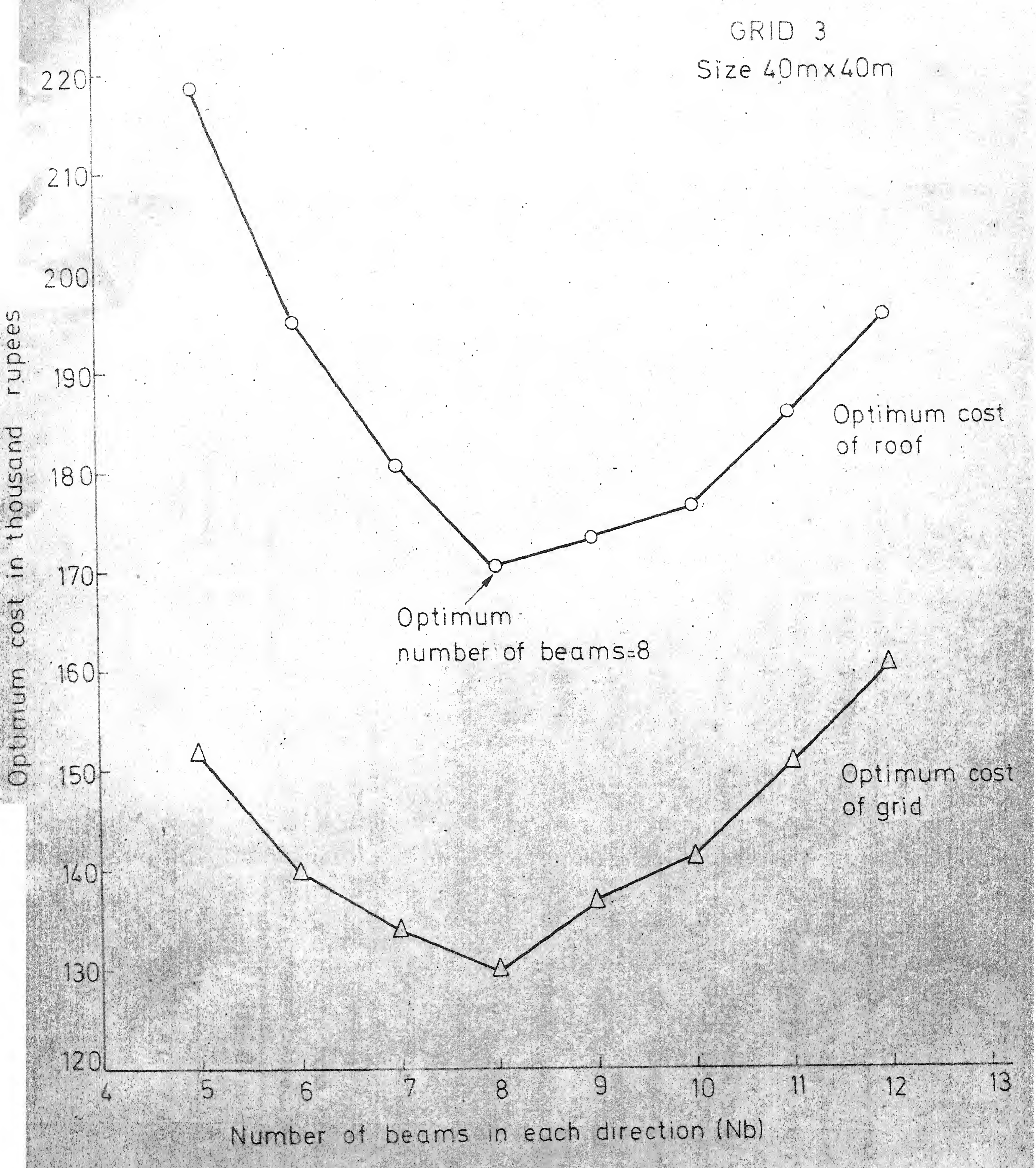


FIG.5.3 VARIATION IN THE OPTIMUM COST OF ROOF AND THE OPTIMUM COST OF GRID WITH RESPECT TO THE CHANGE IN THE NUMBER OF BEAMS IN EACH DIRECTION FOR GRID 3

Table 4

Details at the optimum design points : Grid 1 : size 18.82 M x 18.82 M

No. of beams in each direction Nb	10	9	8	7	6	5	4	3	2
C/C spacing of beams, cms	171.09	188.2	209.11	235.25	268.86	313.66	376.4	470.50	627.33
Slab thickness, cms	10.0	10.0	10.0	10.0	10.0	10.0	10.0	12.0	16.0
Section of beam at optimum b cms	15.08	15.00	15.02	15.04	15.00	15.00	15.08	15.91	20.97
D cms	145.43	122.13	129.39	133.00	113.69	140.90	130.85	157.97	156.06
Total load carried by grid, Kg/m^2	927	782	760	722	620	667	587	614	709
Maximum deflection, cms	0.450	0.661	0.580	0.600	0.860	0.610	0.671	0.540	0.430
Average tension steel % of the C/S	0.387	0.420	0.475	0.510	0.550	0.690	0.725	0.630	0.896
Average compression steel % of the C/S	0.020	0.072	0.070	0.100	0.230	0.190	0.260	0.200	0.570
Average stirrup steel for shear force, cm^2/m	2.73	2.84	3.22	3.50	3.50	4.10	4.50	4.60	8.00
Average stirrup steel for torsion, cm^2/m	0.041	0.110	0.250	0.230	0.420	0.180	0.308	0.252	0.870
Average longitudinal steel for torsion % of the C/S	0.003	0.008	0.018	0.017	0.036	0.012	0.022	0.018	0.047
Cost of slab, Rs.	7,742	7,742	7,742	7,742	7,742	7,742	7,840	9,775	13,750
Cost of grid at optimum, Rs.	23,279	19,394	18,715	17,292	15,083	14,363	12,923	12,289	14,680
Total cost of roof at optimum, Rs.	31,021	27,136	26,457	25,034	22,825	22,105	20,763	22,064	28,430

Table 5

Details at optimum design points : Grid 2 : size 30 M x 30 M

No. of beams in each direction Nb	9	8	7	6	5	4	3
C/C spacing of beams, cms	300.00	333.33	375.00	428.57	500.00	600.00	750.00
Slab thickness, cms	10.0	10.0	10.0	11.0	12.5	15.0	19.0
Section of beam at optimum b cms	15.01	15.00	15.07	15.04	16.67	20.00	25.12
D cms	172.47	189.78	224.25	225.12	246.80	259.00	320.56
Total load carried by grid, Kg/m^2	729	725	747	711	770	852	1040
Maximum deflection, cms	2.25	1.81	1.52	1.50	1.60	1.00	0.75
Average tension steel % of the C/S	0.680	0.642	0.561	0.625	0.651	0.710	0.645
Average compression steel % of the C/S	0.370	0.306	0.220	0.310	0.303	0.341	0.216
Average stirrup steel for shear force, cm^2/m	4.20	4.25	4.09	4.60	6.00	6.34	8.60
Average stirrup steel for torsion, cm^2/m	0.346	0.250	0.127	0.210	0.230	0.372	0.325
Average longitudinal steel for torsion % of the C/S	0.025	0.018	0.009	0.015	0.015	0.020	0.014
Cost of slab, Rs.	19,672	19,672	19,672	22,200	26,271	32,670	43,290
Cost of grid at optimum, Rs.	60,234	56,823	54,192	51,145	53,474	58,291	63,778
Total cost of roof at optimum, Rs.	79,906	76,495	73,864	73,346	79,745	90,961	107,068

Table 6

Details at the optimum design points : Grid 3 : size 40 M x 40 M

No. of beams in each direction Nb	12	11	10	9	8	7	6	5
C/C spacing of beams, cms	307.69	333.33	363.64	400.00	444.44	500.00	571.43	666.67
Slab thickness, cms	10.0	10.0	10.0	10.0	11.0	12.5	14.5	17.0
Section of beam at optimum b cms	15.06	15.06	15.01	15.09	15.09	16.73	19.19	22.30
D cms	327.55	320.49	318.75	336.00	348.38	349.22	341.60	359.64
Total load carried by grid, Kg/m^2	1063	1010	947	945	910	936	982	1037
Maximum deflection, cms	1.65	1.74	1.72	1.51	1.53	1.76	1.62	1.83
Average tension steel % of the C/S	0.513	0.530	0.554	0.529	0.573	0.605	0.653	0.750
Average compression steel % of the C/S	0.150	0.164	0.190	0.165	0.218	0.272	0.342	0.432
Average stirrup steel for shear force, cm^2/m	4.22	4.28	4.45	4.70	4.90	5.80	9.50	8.88
Average stirrup steel for torsion, cm^2/m	0.100	0.095	0.122	0.112	0.123	0.168	0.340	0.525
Average longitudinal steel for torsion % of the C/S	0.007	0.007	0.009	0.008	0.009	0.011	0.013	0.025
Cost of slab, Rs.	34,796	34,796	34,796	36.150	40,450	46,600	55,200	66,800
Cost of grid at optimum, Rs.	160,552	150,716	141,383	137,144	129,991	134,055	139,831	151,932
Total cost of roof at optimum, Rs.	195,348	185,512	176,179	173,294	170,441	180,655	195,031	218,732

Tables 4, 5 and 6 also give the weight carried by the grid per unit area and the maximum deflection in the grid for optimum design points at each spacing studied for Grids 1, 2 and 3 respectively.

The design details at the optimum sections giving the average percentage of tension and compression reinforcements as well as shear and torsional steel are presented in Tables 1, 2 and 3.

5.6 DISCUSSION AND CONCLUSIONS

It is seen from Tables 3, 4 and 5 that for the spacing of beams less than or equal to 375 cms the slab cost is constant for a given overall size of grid. This is because at all the spacings less than or equal to 375 cms the slab thickness is 10 cms and it is provided with minimum specified reinforcement. Thus for $l \leq 375$ cms the variation in the total minimum cost is entirely dependent upon the variation in the minimum cost of grid. This can be observed from Figures 5.1, 5.2 and 5.3 which show a constant difference in the ordinates of the minimum cost of roof and the minimum cost of grid for spacings less than 375 cms. For a spacing greater than 375 cms upto and including 400 cms the slab thickness remains same but the steel

requirement is more than the minimum specified steel. Hence the cost of slab increases slightly. This can be observed from the costs of slab at Nb equal to 5 and 4 for Grid 1 in Table 4 and at Nb equal to 10 and 9 for Grid 3 in Table 6. Thus the dead weight on grid is constant for $l \leq 400$ cms.

When the spacing of beams is greater than 400 cms the slab thickness increases which naturally causes increase in the cost of slab as well as in the dead load on grid. The slab cost increases rapidly as the spacing increases. The variation in the slab thickness and the cost of slab as the spacing becomes greater than 400 cms can be seen in Tables 3, 4 and 5. Figures 5.1, 5.2 and 5.3 also show the variation in the cost of slab for $l \geq 400$ cms from the difference in the ordinates of minimum total cost of roof and minimum cost of grid. Thus the variation in the minimum total cost of roof for $l > 400$ cms is also dependent upon the change in the cost of slab along with the change in the minimum cost of grid.

To study the variation in the cost of grid as the number of beams in each direction changes let us see Figures 5.1, 5.2 and 5.3. The figures show that for $l < 475$ cms the cost of grid increases as the spacing

decreases and for $l > 475$ cms the cost of grid increases as the spacing increases. The requirement of the minimum width of beam increases as l is increased beyond 450 cms and for $l < 450$ cms the minimum width of beam is constant. It is seen from the optimum design points at various spacings that minimum specified width is always reached at the optimum. Thus the larger width requirement for $l > 450$ cms gives more dead weight of the grid which otherwise would have been reduced. Moreover as already explained the dead weight of slab also increases rapidly as the spacing increases beyond 400 cms. Thus the total load carried by grid at larger spacings is more. This is observed in Tables 4, 5 and 6 where the load per unit area carried by grid at the optimum design points is given for Grids 1, 2 and 3 respectively.

Tables 4, 5 and 6 also show that at the optimum designs for $l < 445$ cms the total load carried by the grid increases as the spacing decreases. This is obviously due to the increase in the total number of beams in the grid as the weight of the slab remains constant at all these spacings.

From this it can be concluded that the optimum beam spacing leading to the minimum cost design also turns out to be a minimum weight design.

Regarding the sections of beams at the optimum design for each spacing it can be said that always the minimum specified width is obtained. Though the variation in the depth of sections does not show any fixed trend still it can be observed that at larger spacings relatively more depth is obtained.

Looking at Tables 3, 4 and 5 the variation in the percentage of tension steel shows that at larger spacings the percentage of tension steel is higher. At the spacing decreases the percentage of tension steel also reduces. It was observed from the detail design of the optimum points that at lower spacings only the sections nearer to the supports and those near the centre of the grid require more steel and all other sections are required to be provided with the minimum specified reinforcement. From the torsional reinforcements for optimal designs given in Tables 4, 5 and 6 it is seen that the geometry of the section at the optimum requires very small amount of torsional steel.

The maximum deflections at optimum designs given in Tables 4, 5 and 6 show that the order of maximum deflection at optimum design points is very small.

From the above discussion it can be concluded that the optimum spacing of beams is the one which is

around 400 cms for grids upto a span of 40 M. The minimum cost design at the optimum spacing also turns out to be a minimum weight design. It is also seen that at optimum designs the torsional steel requirement is very small. Further the optimal design gets to the minimum specified width of the beam and the deflections at the optimal designs are of small order.

From the optimum number of beams obtained for the three grids i.e. Grids 1, 2 and 3, an empirical formula for obtaining the optimum number of beams in a grid of given size and of the type considered in the present problem can be suggested. The proposed formula is as follows

$$Nb_{optimum} = \frac{\text{Span of grid in meters}}{5}$$

From the above formula the number of beams at optimum design of Grids 1, 2 and 3 respectively turn out to be 4, 6 and 8 as seen in Tables 1, 2 and 3 respectively.

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